Bayesian evidence for a nonlinear damping model for coronal loop oscillations

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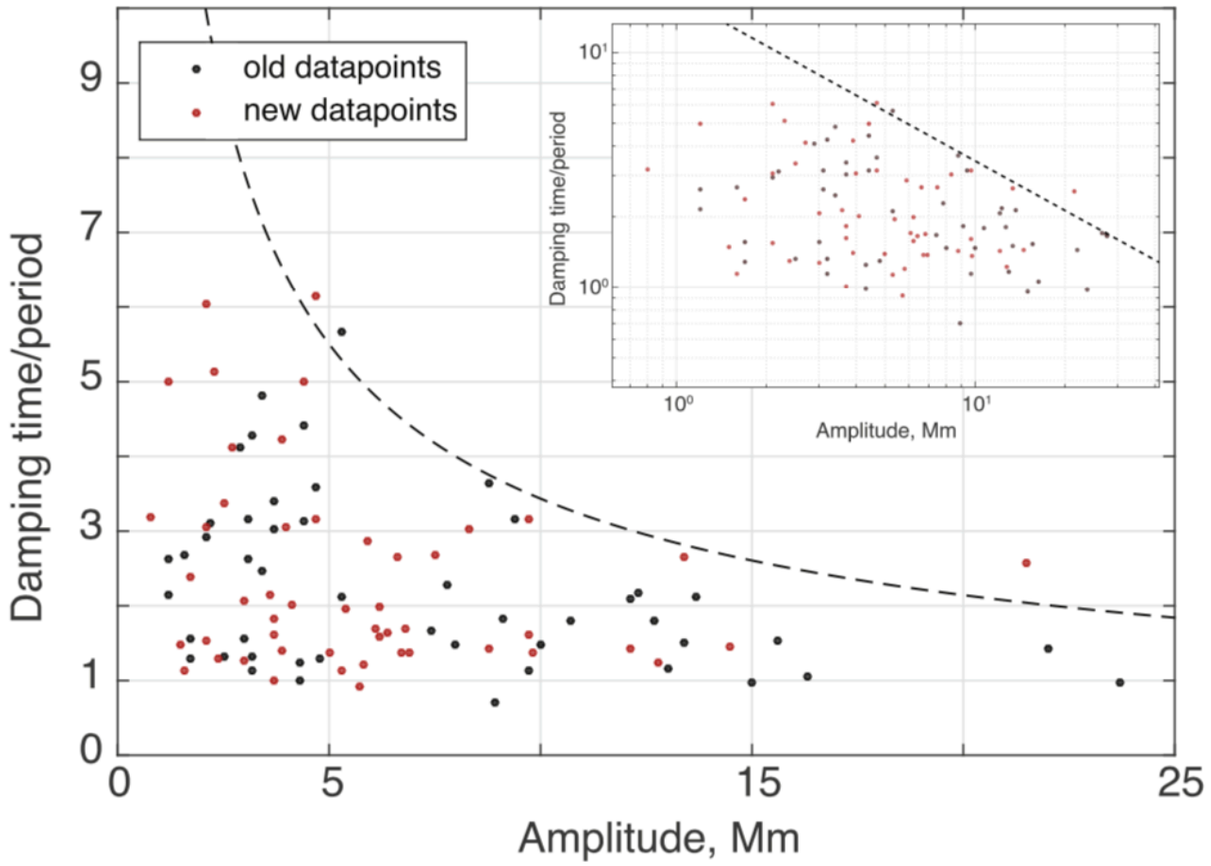
Observations

Empirical relationship between the damping and the oscillation amplitude



Models

Resonant Absorption (RA)

$$\frac{\tau_{\rm d}}{P}\|_{M_{\rm RA}} = \mathcal{F}\,\frac{\zeta+1}{\zeta-1}\,\frac{R}{l}$$

Goossens et al. (2002)

Nonlinear Damping (NL)

Van Doorsselaere et al. (2021)

$$\frac{\tau_{\rm d}}{P}\|_{M_{\rm NL}} = 20\sqrt{\pi}\,\frac{1}{2\pi a}\,\frac{1+\zeta}{\sqrt{\zeta^2-2\zeta+97}}$$

Evidence

Marginal likelihood is a measure of relational evidence

$$p(D|M) = \int_{\boldsymbol{\theta}} p(\boldsymbol{\theta}, D|M)\, d\boldsymbol{\theta} = \int_{\boldsymbol{\theta}} p(D|\boldsymbol{\theta}, M)\, p(\boldsymbol{\theta}|M)\, d\boldsymbol{\theta}$$

$$p(D\,|\,M_{\mathrm{NL}}) \;\&\; p(D\,|\,M_{\mathrm{RA}})$$

Bayes Factor is a measure of relative evidence

$$B_{\mathrm{NLRA}} = 2\log\frac{p(D\,|\,M_{\mathrm{NL}})}{p(D\,|\,M_{\mathrm{RA}})} = -\,B_{\mathrm{RANL}}$$

Kass & Raftery (1995)

Bayes factor	Evidence
0 - 2	inconclusive
2 - 6	positive
> 6	strong
> 10	very strong

Application to SDO/AIA loop oscillations

Bayes factor for 101 oscillation events in the catalog by Nechaeva et al. (2019)

i	Event ID	Loop ID	P [min]	τ_D [min]	τ_D/P	η [Mm]	B_{NLRA}
1	1	1	3.42 ± 0.06	5.34 ± 1.12	1.56 ± 0.33	1.7	0.4
2	1	2	4.11 ± 0.05	10.76 ± 2.79	2.62 ± 0.68	1.2	1.4
3	3	1	2.46 ± 0.03	8.8 ± 1.8	3.58 ± 0.73	4.7	3.4
4	3	2	3.62 ± 0.08	4.12 ± 0.47	1.14 ± 0.13	9.7	2.2
5	4	1	2.29 ± 0.03	7.18 ± 1.5	3.14 ± 0.66	4.4	3.5
6	4	2	3.47 ± 0.03	$7.44 \pm 1.$	2.14 ± 0.29	1.2	1.0
7	7	1	1.69 ± 0.02	7.23 ± 1.3	4.28 ± 0.77	3.2	4.1
8	8	1	3.74 ± 0.07	$10. \pm 1.$	2.67 ± 0.27	1.6	2.5
9	9	1	5.14 ± 0.17	5.09 ± 0.98	0.99 ± 0.19	4.3	0.9
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94	84	1	3.78 ± 0.33	15.6 ± 8.3	4.13 ± 2.23	2.7 ± 1.0	2.6
95	87	1	5.94 ± 0.32	25.1 ± 10.8	4.23 ± 1.83	3.9 ± 1.0	2.7
96	87	2	9.24 ± 0.62	12.7 ± 4.6	1.37 ± 0.51	6.9 ± 2.2	2.1
97	88	1	9.38 ± 0.19	13.0 ± 5.4	1.39 ± 0.58	5.0 ± 0.4	1.8
98	88	3	13.75 ± 1.18	15.5 ± 6.5	1.13 ± 0.48	5.3 ± 1.8	1.5
99	90	1	9.32 ± 0.31	8.6 ± 2.4	0.92 ± 0.26	5.7 ± 1.4	1.2
100	92	1	6.51 ± 0.31	9.1 ± 2.4	1.40 ± 0.37	3.9 ± 1.3	1.6
101	93	1	8.32 ± 0.10	21.4 ± 4.9	2.57 ± 0.59	21.5 ± 2.4	-6.2

$B_{NLRA} > 0$

Evidence if favour of nonlinear damping

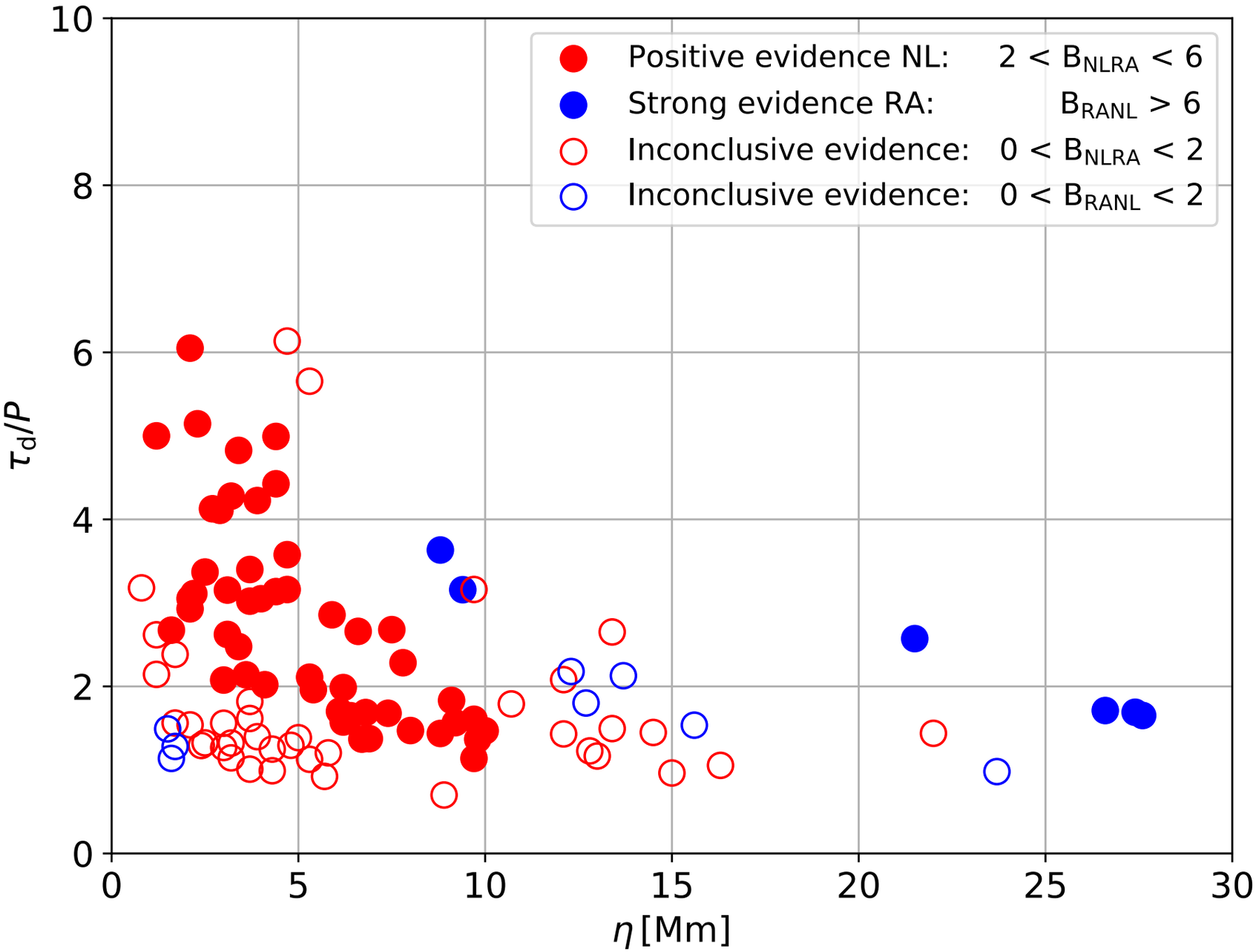
Bayes factor	Evidence
0 - 2	inconclusive
2 - 6	positive
> 6	strong
> 10	very strong

$B_{NLRA} < 0$

Evidence if favour of resonant damping

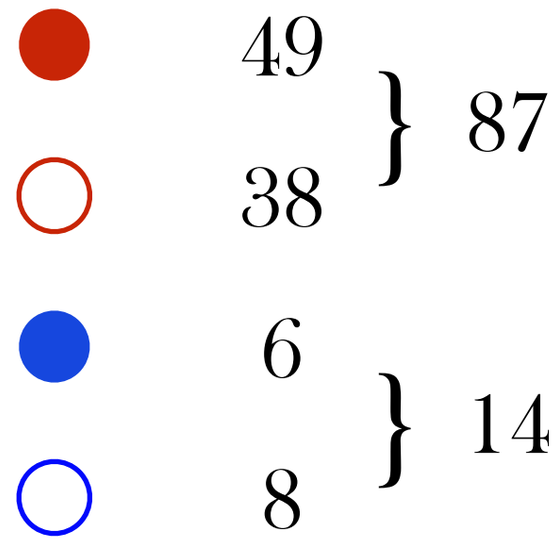
Application to SDO/AIA loop oscillations

(bayesian) results for all 101 oscillating loops



(frequentist) statistics

cases



The nonlinear damping model offers a plausible explanation for the observed properties of damped transverse coronal loop oscillations